N71-35901

NASA TECHNICAL TRANSLATION

NASA TT F-13,736

THE PROBLEM OF THE GENERATION OF PARTICLES IN THE CLASSICAL MECHANICS

Yu. A. Rylov

Translation of "Problema Rozhdeniya Chastits v Klassicheskoy Mekhanike," Institute for Space Research, Academy of Sciences, USSR, Report Pr-21, 1969, pp. 5-21

CASE FILE COPY

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546

THE PROBLEM OF THE GENERATION OF PARTICLES IN THE CLASSICAL MECHANICS

Yu. A. Rylov

ABSTRACT: It was shown in [2], that the phenomenon of generation of particles could be described in terms of classical relativistic mechanics. A classical "generating pairs" field (q) was introduced. A sense of this field is cleared up in the paper. This field is shown to be treated as a classical equivalent of a certain "quantum field". The limit transition from the quantum mechanics to the classical one corresponds to the disappearance of the classical field f (q) conserving the phenomenon of generation of particles.

A new method of receiving quantum relativistic equations is described.

I. As is known, classical relativistic mechanics does not allow the universal path of a particle to turn backwards in time. In other words, classical mechanics can not describe the appearance of the origin and annihilation of particle-antiparticle pairs. This is connected with the fact that the four components of the canonical ensemble describing the state of the particle are not independent, but are connected by a certain relation. For a free particle this relation has the form: /5*

$$p_i g^{ik} p_k = m^2 c^2. \qquad (1)$$

where g^{ik} is a metric tensor, m is the mass of the particle, and c is the speed of light. The latin indexes here and below require the values 0, I, 2, 3. In repeating indexes the summation from 0 to 3 is carried out. The metric tensor takes the form

* Numbers in the margin indicate pagination in the foreign text.

$$g_{ik} = \begin{Vmatrix} c^2 & 0 & 0 & 0 \\ 0 & -I & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -I \end{Vmatrix} \qquad g^{ik} = \begin{Vmatrix} \bar{c}^2 & 0 & 0 & 0 \\ 0 & -I & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -I \end{Vmatrix} \tag{2}$$

Equation (1) describes a two-cavity hyperboloid in the field of the canonical ensembles ρ_i. The allowable values of the ensembles lie in the hyperboloid.

In classical mechanics the state of a particle changes continuously, therefore the transition from one field of the hyperboloid to another is impossible and, consequently it is impossible to change the sign of the time component of the canonical ensemble ρ_0. To turn the universal path of a particle backwards in time (i.e., for the annihilation of a pair) it is necessary to change the sign of ρ_0, which is impossible in the framework of classical mechanics. A calculation of the electromagnetic field alters the form of relation (1), but does not eliminate the relation between the canonical ensembles. In this case the possible states form two unconnected regions and the production and annihilation of pairs are impossible. /6

2. From a formal point of view, the existence of a relation of the type (1) between canonical ensembles is a consequence of the fact that only the universal path of a particle has a physical sense. Let the four functions $q^i = q^i(\tau)$ be the parametric representation of the universal path of a particle. Here, τ represents a parameter of the universal path of a particle. It is obvious that the form of the path does not depend on the choice of the parameter τ. In particular the path is not changed on transformation of the parameter τ.

$$\tau \to \tau' = \varphi(\tau), \tag{3}$$

where $\varphi(\tau)$ is an arbitrary monotonic function of its own argument. As long as only the universal path of a particle itself has a physical sense, but not its parameters, then equations describing the universal path must not depend on the choice of the parameter and must be invariant relative to transformation of (3). In particular, operation (1) /7

$$S = \int L \, d\tau, \quad L = L(q^i(\tau), \dot{q}^\kappa(\tau)), \quad \dot{q}^i \equiv dq^i/d\tau \tag{4}$$

must be invariant with respect to the transformation (3). For this it is necessary that L be a homogeneous function of the first order of $\dot{q}^i$ i.e.,

$$L(q^i, a\dot{q}^\kappa) = aL(q^i, \dot{q}^\kappa), \tag{5}$$

where a is an arbitrary value. Differentiating (5) with respect to a and setting $a = 1$, we obtain

$$\dot{q}^i \frac{\partial L}{\partial \dot{q}^i} - L = 0 \tag{6}$$

As usual we determine the canonical ensemble p_i by the relation

$$p_i = \partial L / \partial \dot{q}^i, \tag{7}$$

and the Hamiltonian function H by

$$H = p_i \dot{q}^i - L. \tag{8}$$

The function H, by using relation (7) must be represented as

$$H = H(q, p), \quad q = (q^0, q^1, q^2, q^3),$$
$$p = (p_0, p_1, p_2, p_3).$$

From (6), (7), and (8) it follows that

$$H = 0 , \tag{9}$$

and the four relations (7) are forbidden single-valuedly relative to $\dot{q}^i$. Indeed, suppose in some region changes of the ensembles p_i of relation (7) allow a solution of the form

$$\dot{q}^i = f^i(q, p). \tag{10}$$

In virtue of (5), $\partial L / \partial \dot{q}^i$ are homogeneous functions of zero order from $\dot{q}^i$ and

$$\dot{q}^i = a f^i(q, p), \tag{11}$$

where a is an arbitrary value, is also a solution of equation (7). Substituting (11) into (6), we obtain the condition in the canonical ensembles p_i / a (in this a cancels out). /8

Thus the canonical ensembles p_i are dependent, with the connection between them determined by the form of the Lagrangian (7), but the fact of the existence of the connection is a consequence of the invariance of operation (4) relative to the transformation of (3). In particular, the existance of equation (1) is a consequence of the invariance of the operation relative to (3).

3. It seems that the invariance relative to (3) makes it impossible to describe the appearance of the production and annihilation of pairs of particles in the frameworks of classical mechanics. It is possible, however, to examine the operation of the form [2]:

$$S = \int L\,d\tau, \quad L = -\sqrt{m^2c^2 \dot{q}^i g_{ik} \dot{q}^k} - \beta f(q), \qquad (12)$$

where $f(q)$ is a fixed scalar field and β is a non-negative constant. The operation (12) is non-invariant relative to (3), and essentially, (12) already describes another mechanics, where all four components of the canonical ensemble p_i are independent. In this mechanics, the universal path of a particle can turn back in time, i.e., it is possible to describe the production and annihilation of a pair of particles. We will now examine the usual relativistic mechanics as the limit of this mechanics with $\beta \to 0$. then in the limit, the invariance of the operation relative to the transformation of (3) is reestablished, but the possibility of the production $f(q) = 0$ annihilation of pairs is retained. In the limit of these points of the field of events, where $f(q) = 0$, breakdowns can arise in the universal path and /9 it can turn back and annihilate particles. As this takes place, the field $f(q)$ acquires several mystical characteristics. Though it is absent in the Lagrangian boundary, it nevertheless can generate and annihilate particles.

4. Comment The ability of the field $f(q)$ to generate pairs is connected with the introduction of it to a radical. If, assuming β to be small, we expand (12) by powers of β and limit it to powers not greater than the first, the Lagrangian obtained

$$L = -mc\gamma + \frac{\beta}{2mc}\,\frac{f(q)}{\gamma}, \quad \gamma = \sqrt{\dot{q}^i g_{ik} \dot{q}^k}$$

does not allow a solution in the form of a universal path turning back in time, i.e., the ability to generate pairs is lost.

5. To clear up the sense of the field $f(q)$ we attempt to make the Lagrangian (12) invariant relative to (3), not passing the limit $\beta \to 0$. We examine an ensemble of like particles of mass M. Each of which is described by the Lagrangian (12). We will assume the special starting conditions, that the universal paths of these particles do not intersect each other and that they would tightly fill some region of the field of events. We will characterize the universal path of each particle by the three Lorentzian-invariant parameters ξ_1, ξ_2, ξ_3. We further introduce the designation $\xi_0 = \tau$, $\xi = \{\xi_0, \xi_1, \xi_2, \xi_3\}$. Operation (12) for an ensemble of like particles can be described in the form

$$S = S[q(\xi)] = \int L\, d^4\xi, \qquad d^4\xi = d\xi_0\, d\xi_1\, d\xi_2\, d\xi_3,$$
$$L = -\sqrt{m^2c^2 \frac{\partial q^i}{\partial \xi_0} g_{ik} \frac{\partial q^k}{\partial \xi_0}} - \beta f(q), \tag{13}$$

where $q^i = q^i(\xi)$ are unknown functions of the four arguments ξ_i, with ξ_α ($\alpha = 1, 2, 3$) introduced into (13) only as variables. Writing (13) as an integral with respect to the variables ξ_i incorporates into itself the assumption of the special, continuous selection of the initial conditions of the particles constituting the ensemble. The problem of finding the universal paths of the particles of the ensemble is that of locating the extrema of functional (13). The extrema of functional (13) are the four functions /10

$$q^i = q^i(\xi). \tag{14}$$

6. We will allow that in some regions of change of ζ_i

$$J^{-1} = \frac{\partial(q^0, q^1, q^2, q^3)}{\partial(\zeta_0, \zeta_1, \zeta_2, \zeta_3)} \neq 0, \tag{15}$$

then in this region of equation (14) capable of solution relative to ζ_i :

$$\zeta_i = \zeta_i(q),$$

and the operation S can be written in the form

$$S = S_1[\zeta(q)] = -\int d^4q \sqrt{m^2c^2 \frac{\partial J}{\partial \zeta_{0,i}} g_{ik} \frac{\partial J}{\partial \zeta_{0,k}} - \beta f(q) J^2}, \tag{16}$$

$$d^4q = dq^0 dq^1 dq^2 dq^3,$$

where $\zeta_i = \zeta_i(q)$, $\zeta_{i,k} \equiv \partial \zeta_i / \partial q^k$, J is the Jacobian, determined by (15) and under consideration as a function of $\zeta_{i,k}$. In transforming (13) to the form (16), we use the obvious relations

$$d^4\zeta = J d^4q, \tag{17}$$

$$J \frac{\partial q^i}{\partial \zeta_0} = \frac{\partial(q^i, \zeta_1, \zeta_2, \zeta_3)}{\partial(\zeta_0, \zeta_1, \zeta_2, \zeta_3)} \frac{\partial(\zeta_0, \zeta_1, \zeta_2, \zeta_3)}{\partial(q^0, q^1, q^2, q^3)} = \frac{\partial J}{\partial \zeta_{0,i}}, \tag{18}$$

while at the same time in (18) J is considered as a function of $\zeta_{i,k}$. /11

During the execution of (15) as long as the variations are unambiguously determined by the variation $\delta\zeta_i(q)$ and vice versa, then the extrema of functional (13) $S = S[q(\zeta)]$ coincide with those of functional (16) $S = S_1[\zeta(q)]$. Since we are only concerned with the sense of the field $f(q)$, we need not make a firm equivalence of the variational problem with functionals (13) and (16). We will ignore condition (15) which is an inequality, and is not fulfilled, generally speaking, only in special cases. Ignoring (15), we will speak about the formal equivalence of the variational problems with functionals (13) and (16).

7. We introduce the criterion

$$n = \frac{1}{c}\sqrt{\frac{\partial f}{\partial \zeta_{0,i}}\, g_{ik}\, \frac{\partial f}{\partial \zeta_{0,k}}} \tag{19}$$

and rewrite (16) in the form

$$S = \int L\, d^4q, \qquad L = L_0 + L_1, \tag{20}$$

$$L_0 = -\sqrt{m^2c^4n^2 - \beta f(q) J^2}, \tag{21}$$

$$L_1 = \mu\left(n - \frac{1}{c}\sqrt{\frac{\partial f}{\partial \zeta_{0,i}}\, g_{ik}\, \frac{\partial f}{\partial \zeta_{0,k}}}\right). \tag{22}$$

Here (20) is the functional $S = S_2 [\xi(q), n(q), \mu(q)]$ and formally has the same extrema as (16) and (13) [3], i.e., setting the variation derivatives of (20) with respect to $\xi_i(q), n(q), \mu(q)$ to zero, we obtain equations which are formally equivalent to those, obtained by setting the variation derivatives of (13) with respect to $q^i(\xi)$ to zero. The variation problem with functional (20) is obtained from (16) by the addition of condition (19) with the Lagrangian factor m. /12

Operation (20), as is easily verified, is not invariant relative to the transformation (3) (recall that $\tau = \xi_0$). The non-invariance results, because of the term $-\beta f(q) J^2$, which contains the values $\xi_{0,i} = \partial\tau/\partial q^i$. The other terms, among which are $\partial J / \partial \xi_{0,i}$, contain neither ξ_0, nor derivatives of ξ_0. The simplest possibility to obtain the invariance of S relative to the transformation of (3) consists of eliminating the term $-\beta f(q) J^2$, for example, by setting $\beta \to 0$. Indeed this possibility was used in [2]. However, this is not the only possibility. It is not necessary to eliminate the term $-\beta f(q) J^2$. It is sufficient to make it invariant relative to (3). We substitute $\beta f(q) J^2$ into the expression

$$\frac{\hbar^2 c^2}{4} \frac{\partial n}{\partial q^i} g^{ik} \frac{\partial n}{\partial q^k}$$

the latter being independent of $\hbar$. Here $\hbar$ is Planck's constant. We eliminate f from (21), setting

$$\beta f J^2 = \frac{\hbar^2 c^4}{4} \frac{\partial n}{\partial q^i} g^{ik} \frac{\partial n}{\partial q^k} \tag{23}$$

After this the operation becomes invariant relative to (3) and the universal paths of the particles in the ensemble find the usual sense, since they transpose depending on the choice of parameters in them.

Introducing:

$$\lambda = \frac{\hbar}{2mc} \tag{24}$$

and using (23), we rewrite (20) - (22) in the form

$$S = S_3[\zeta(q), \mu(q), n(q)] = \int L\, d^4q, \quad L = L_0 + L_1, \tag{25}$$

$$L_0 = -mc^2 K, \quad K = \sqrt{n^2 - \lambda^2 n_i g^{ik} n_k}, \quad n_i \equiv \partial n / \partial q^i \tag{26}$$

/13

$$L_1 = \mu\left(n - \frac{1}{c}\sqrt{\frac{\partial \zeta}{\partial \zeta_{0,i}} g_{ik} \frac{\partial \zeta}{\partial \zeta_{0,k}}}\right). \tag{27}$$

8. Before proceeding to the formal consequences of the variational problem with functional (25), we will inquire into the sense of the terms introduced into the Langrangian (25), if we assume that in the interval of change $\Delta\zeta_\alpha = 1 \quad (\alpha = 1, 2, 3)$ of the variables ζ_α there is one particle of the ensemble, that acts in (27) the vector $\partial\zeta / \partial\zeta_{0,i}$ can be interpreted as a four-vector stream of particles of the ensemble. This can be seen from an analysis of equation (18). In particular, the time component $\partial\zeta / \partial\zeta_{0,0}$ is the density of the number of particles in the system of coordinates q^i. The total number of particles in the ensemble

$$N = \int_{-\infty}^{\infty}\int\int\int \frac{\partial \zeta}{\partial \zeta_{0,0}}\, d^3q = \int_{-\infty}^{\infty}\int\int\int \frac{\partial(\zeta_1, \zeta_2, \zeta_3)}{\partial(q^1, q^2, q^3)}\, d^3q = \prod_{\alpha=1}^{3} \Sigma_\alpha \tag{28}$$

Here Σ_α signifies an interval of change with values in the whole field of events. At the same time, it is assumed that $\partial\zeta / \partial\zeta_{0,0} > 0$ everywhere. From a determination of n with the help of relation (19) it

follows that n is the density of the number of particles of the ensemble in a fundamental frame of reference. Further, from (26) it follows that the first term in L_0 $mc^2 n$ is the energy density of the particles of the ensemble in a fundamental frame of reference, with the energy given to one particle being mc^2 i.e., this term describes the fundamental energy of the particle of the ensemble. The second member in L_0 has the form $0{,}5\hbar c\sqrt{-n_i g^{ik} n_k}$. It is also the energy density in a fundamental frame of reference. This member /14 describes the internal energy--the energy of the interaction of the particles in the ensemble. This energy depends on the inhomogeneity of the distribution of the particles in the ensemble. The greater the inhomogeneity of the ensemble, i.e., the larger is the density gradient of the particles $n(q)$, the larger is the role played by the member which describes the energy of the interaction of the particles in the ensemble. This condition is reminiscent of gas dynamics or Brownian motion, where the appearance of a density gradient of the particle leads to the operation of statistical mechanics, and gives rise to diffusion and the appearance characteristic of gas-dynamic forces of the pressure type. The internal energy density appears to have a universal value in the sense that it depends only on the density gradient of the particles, and not on characteristics of the particles of the ensemble, such as the mass of the particles m [1]. The universal constant $\hbar c$ serves as the proportionality constant. However, the effect of the internal energy on the motion of the particles depends on the relation between the terms $mc^2 n$ and $0{,}5\hbar c\sqrt{-n_i g^{ik} n_k}$. The extent of this effect can be evaluated from the value of $\lambda^2 n_i g^{ik} n_k / n^2$ where λ is given by relation (24) and is half the Compton wave length of the particles. We see that, all other things being equal, the term $0{,}5\hbar c\sqrt{-n_i g^{ik} n_k}$ has a larger effect on the light particles. In this respect our ensemble is very reminiscent of an ensemble of Brownian particles, where the light particles

---

[1] An Analogous situation exists in the kinetic theory of gases, where the average energy of a molecule is equal to $3kT/2$ and does not depend on the mass of the molecule, but only on the properties of the medium as a whole.

are more susceptible to the effect of the medium, and the more decisive factor for the motion of the ensemble is the density gradient of the particles. In general, the ensemble under consideration can be understood as a unique ensemble of relativistic particles. Each of the particles interacts with the "medium" (the ether) in an unpredicted way. Planck's constant characterizes the degree of this interaction. The action of each separate particle is unpredictable (as is the case with a Brownian particle), but the action of an ensemble of such particles can be predicted, of course, statistically, i.e., in the same manner as with Brownian particles. At the same time $\mathfrak{z}_\alpha$, appearing as the Lagrangian coordinates of an element of three-dimensional space and containing a large number of relativistic Brownian particles, describe the motion of this element of three-dimensional space, i.e., they describe the motion of the particles only in the medium. The true motion of each particle is random and is superimposed on the average motion. The term $0{,}5\,\hbar c\sqrt{-n_i g^{ik} n_k}$ determines the effect of the random motion on the medium. Of course, the random motion has an effect on the medium only where a density gradient exists. One can conceive of $0{,}5\,\hbar c\sqrt{-n_i g^{ik} n_k}$ being the energy density of random motion, although perhaps it is only a part of the energy of random motion, namely that part which can be transformed into energy of regulated motion. /15

We notice, finally, that the above mentioned ensemble of particles can be looked at as some hydrodynamics [4]. In so doing, the "average" particles of the ensemble must be identified with "the liquid particles", and $\mathfrak{z}_\alpha$ will be the Lagrangian coordinates of the elements of the liquids, while q^i are the corresponding Euler coordinates. The pressure tensor has a rather exotic form and is proportional to $\hbar^2$. Nevertheless all of this formally looks like hydrodynamics. With respect to L_1 from (27) we note that L_1 has no special physical sense and is a way to evaluate relation (19) in the framework of the variational problem. L_1 becomes zero because of the equations of motion. /16

Further it will be shown that the equations of motion, obtained as a result of the modification of (25) with respect to n, μ and ζ_i, are a particular case of the Klein-Gordon equation (approximately), and in a nonrelativistic approximation the Schroedinger equarion can be obtained from them.

9. Modifying (20) we have

$$\frac{\delta S}{\delta n} = -mc^2\left[\frac{n}{K} + \lambda^2 \frac{\partial}{\partial q^i}\left(\frac{g^{ik} n_k}{K}\right)\right] + \mu = 0, \tag{29}$$

$$\frac{\delta S}{\delta \mu} = n - \frac{1}{c}\sqrt{\frac{\partial J}{\partial \zeta_{0,i}} g_{ik} \frac{\partial J}{\partial \zeta_{0,k}}} = 0, \tag{30}$$

$$\frac{\delta S}{\delta \zeta_i} = \frac{1}{c^2}\frac{\partial}{\partial q^i}\left\{\frac{\mu}{n}\frac{\partial^2 J}{\partial \zeta_{0,j}\partial \zeta_{i,\ell}} g_{js} \frac{\partial J}{\partial \zeta_{0,s}}\right\} = 0 \tag{31}$$

We introduce the notation

$$u^k = \frac{1}{nc}\frac{\partial J}{\partial \zeta_{0,k}}, \qquad u_i \equiv g_{ik} u^k. \tag{32}$$

Here u^i is the four-speed of the average motion of the particles of the ensemble and because of (30), it satisfies the relation

$$u^i g_{ik} u^k = I. \tag{33}$$

Using (32) and the identity

$$\frac{\partial}{\partial q^\ell}\frac{\partial^2 J}{\partial \zeta_{0,k}\partial \zeta_{i,\ell}} = 0, \tag{34}$$

We write (31) in form

$$\frac{\partial^2 J}{\partial \zeta_{0,k}\partial \zeta_{i,\ell}}\frac{\partial}{\partial q^\ell}(\mu u_k) = 0. \tag{35}$$

Further, we multiply (35) by $\zeta_{i,s}$ and sum over i. Then, because of the identity /17

$$\frac{\partial^2 J}{\partial \zeta_{0,k}\partial \zeta_{i,\ell}}\zeta_{i,s} = \delta_s^\ell \frac{\partial J}{\partial \zeta_{0,k}} - \delta_s^k \frac{\partial J}{\partial \zeta_{0,\ell}} \tag{36}$$

we obtain

$$n u^{\kappa} \{ \partial_s (\mu u_{\kappa}) - \partial_{\kappa} (\mu u_s) \} = 0, \qquad \partial_i \equiv \frac{\partial}{\partial q^i} . \tag{37}$$

The three independant equations of (37) along with the identity

$$\frac{\partial}{\partial q^i} (n u^i) = \frac{\partial}{\partial q^i} \left(\frac{\partial \mathcal{L}}{\partial \beta_{0,i}} \right) \equiv 0 \tag{38}$$

and relations (26) and (29) constitute a system of equations for the 5 independant variables n, u^i, μ.

10. We will examine a potential solution of (37)

$$\mu u_{\kappa} = c \varphi_{\kappa} , \qquad \varphi_{\kappa} \equiv \partial_{\kappa} \varphi , \tag{39}$$

Where φ is some function of the coordinates q^i. The equations of (37) are satisfied by (39) with any choich of φ. We introduce the notation

$$\rho = \frac{n m c^2}{\mu} , \qquad \psi = \sqrt{\rho} \exp(i \varphi / \hbar) \tag{40}$$

Now we assume that n changes little within the Compton wave length λ, i.e.,

$$\left| \lambda^2 n_i g^{i\kappa} n_{\kappa} \right| \ll n^2 . \tag{41}$$

Then (29) can be expanded into a series of degree λ^2 limited only to terms of an order no higher than λ^2. We will now show that in this case, because of equations (29) and (38), the function ψ, determined by formula (40) to an accuracy of λ^2, is a solution of the Klein-Gordon equation

$$\hbar^2 \partial_i \partial^i \psi + m^2 c^2 \psi = 0, \qquad \partial^i \equiv g^{ik} \partial_k . \tag{42}$$

From (29) to an accuracy of λ^2 we have /18

$$\mu^2 = m^2 c^4 \left(1 + 4\lambda^2 n^{-\frac{1}{2}} \partial_i \partial^i \sqrt{n}\right) \tag{43}$$

On the other side, from (33) and (39) it follows that

$$\mu^2 = c^2 \varphi_k \varphi^k, \qquad \varphi^k \equiv g^{ki} \varphi_i . \tag{44}$$

From the first relation of (40) it follows that n in (43) to an accuracy of λ^2 can be substituted for ρ. As a result (43) and (44), because of (24) give

$$m^2 c^2 + \hbar^2 \rho^{-\frac{1}{2}} \partial_i \partial^i \sqrt{\rho} - \varphi_k \varphi^k = 0 . \tag{45}$$

Substitution of n from the first relation of (40) into (38) gives

$$\partial_i (\rho \varphi^i) = 0 . \tag{46}$$

Multiplying (45) by $\sqrt{\rho}\exp(i\varphi/\hbar)$ and (46) by $i\hbar\rho^{-\frac{1}{2}}\exp(i\varphi/\hbar)$ and adding them, we obtain one complex equation (42) for ψ.

II. As is known (see [5] for example) Schroedinger's equation can be obtained as the nonrelativistic limit ($c \to \infty$) of the Klein-Gordon equation. In this case the difference between the Klein-Gordon equation (42) and equations (29), (37), and (38), having the array $(\lambda^2 n_i g^{ik} n_k / n^2)^2$, tends toward zero in the limit $c \to \infty$, since because of (24) $\lambda \to 0$, i.e., the potential solution (39) precisely satisfies the Schroedinger equation in the nonrelativistic limit.

In this manner an examination of the ensemble of particles and the identification of "forming pairs" of the field $f(q)$, with some statistical interaction in the ensemble, leads to the usual quantum mechanics in a nonrelativistic approximation. In so doing in the nonrelativistic case the sense of $n = \rho = |\psi|^2$, as the probability density of detection of the frequency, follows immediately from the properties of the statistical ensemble. In the nonrelativistic case equations (29), (37), and (38) in conjunction with the limited (39) are equivalent to Schroedinger's equation for a free particle. It is necessary to say this about the relativistic case. The Klein-Gordon equation is a linear "quasi-classical" approximation to equations (29), (37), and (39). The quasi-classical" quality becomes apparent by using the condition of (41), which signifies a small change of n within an order of the Compton wave length. /19

12. Equations (29), (37), and (38), generally speaking, do not assume the principle of linear superposition, which is a basis of contemporary quantum mechanics and which is used both in nonrelativistic quantum mechanics and in the relativistic quantum field theory. From the point of view of equations (29), (37), and (38) the linearity appears only as a result of the quasi-classical approximation. This result is a convenient one, but not principal. The examination of the statistical ensemble allows the possibility of generalizing quantum mechanics in an unexpected direction--the direction of repudiating the principle of linear superposition.

The field $f(q)$ in the Lagrangian (12), the ability to create pairs, can be interpreted as a certain classical equivalent of the quantum field, with the limiting case being when in (12) $\beta \to 0$, which corresponds to the limiting transition from quantum mechanics to classical mechanics ($\hbar \to 0$). With this limiting transition to classical mechanics, the possiblity of relativistic quantum mechanics to describe the creation of pairs of particles preserved. /20

We take this opportunity to thank E.E. Shnolyu for his discussions of this paper and his comments.

REFERENCES

1. Fok, V.A., *Izv. AN SSSR OMEN*, Vol. 551, 1937 (see also V.A. Fok, *Raboty po Kvantovoy Teorii Polya* [Works on Quantum Field Theory], Leningrad Univ. Press, 1957) /21
2. Rylov, Yu. A., Preprint of the Institute of Cosmic Investigations, Acad. Sci., USSR, Report 3, 1969
3. Currant, R., and D. Gilbert, *Methods of Mathematical Physics*, "GITTL" Press, Part 1, Vol. 1, No. 206, 1951.
4. Rylov, Yu. A., *Trudy Vsesoyuznoy Konferentsii po Fizike Kosmicheskikh Luchey* [Works of the All-Union Conference on the Physics of Cosmic Rays], FIAN, P.N. Lebedev Physics Institute of the USSR Academy of Science, Part 1, 3rd Edition P. 47, 1969.
5. Sommerfield, A., *The Structure of Atoms and Spectra*, "GITTL" Press, Part II, Vol. 2, 1956.

Translated for the National Aeronautics and Space Administration under contract No. NASw-2037 by Techtran Corporation, P.O. Box 729, Glen Burnie, Maryland, 21061, Gary P. Rabold, Ph.D., Translator.